

How to write a two-column proof — read this before every proof in Chapters 2 and beyond.

Anatomy of a Two-Column Proof

GIVEN — the facts you start with. They are always **Statement 1**, with reason “Given.”

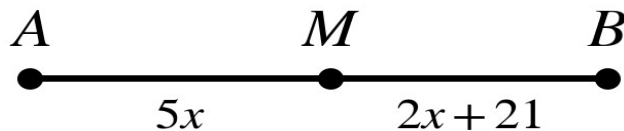
PROVE — the goal. It is always the **LAST** statement.

Each statement must follow logically from earlier statements, and every statement needs a reason: a given, a definition, a property, a postulate, or a theorem.

Work the two ends toward the middle: write the given first, the goal last, and ask “what connects them?”

Worked Example

Given: M is the midpoint of segment AB, $AM = 5x$, $MB = 2x + 21$.
Prove: $x = 7$.



Statements	Reasons
1. M is the midpoint of segment AB	1. Given
2. $AM = MB$	2. Definition of midpoint
3. $5x = 2x + 21$	3. Substitution Property of Equality
4. $3x = 21$	4. Subtraction Property of Equality
5. $x = 7$	5. Division Property of Equality

Five Habits of Good Proof-Writers

- Mark the diagram first — ticks, right angle boxes, and given measures.
- Turn every definition into an equation as soon as you can (midpoint $\rightarrow AM = MB$).
- One idea per line. If a statement does two things, split it.
- Reasons are names, not explanations — “Subtraction POE,” not “I subtracted.”
- Check the last line: it must match the Prove exactly.

Reason Bank — Properties & Definitions

Properties of Equality: Addition • Subtraction • Multiplication • Division • Substitution • Distributive • Reflexive • Symmetric • Transitive

Properties of Congruence (segments & angles): Reflexive • Symmetric • Transitive

Definitions: midpoint • segment bisector • angle bisector • right angle • perpendicular • complementary • supplementary • linear pair • vertical angles • congruent ($\cong \leftrightarrow$ equal measure)

Reason Bank — Postulates & Theorems

Postulates: Segment Addition • Angle Addition • Linear Pair (linear pair $\rightarrow$ supplementary)

Theorems: Right Angles Congruence • Congruent Supplements • Congruent Complements • Vertical Angles Congruence

Now You Try

Given: Ray BD bisects $\angle ABC$, $m\angle ABD = (3x + 8)^\circ$, $m\angle DBC = (5x - 6)^\circ$.
Prove: $x = 7$.

Statements	Reasons
1. Ray BD bisects $\angle ABC$	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

(Answers: def. of angle bisector $\rightarrow$ def. of $\cong$ angles $\rightarrow$ substitution $\rightarrow$ subtract $3x$, add 6 $\rightarrow$ divide by 2. Check: $3(7)+8 = 29$ and $5(7)-6 = 29 \checkmark$)